

1. Find a counterexample to Brouwer's fixed point theorem when the domain S is not closed. E.g., find a continuous map f from $[0, 1)$ to itself with no fixed point.

2. Prove the following grid version of Sperner's Lemma:

Consider a 3-coloring over $\{0, 1, \dots, n\} \times \{0, 1, \dots, n\}$. It is said to be *proper* if

- (a) The color of $(0, i)$ is 'red' for all $i \in \{0, 1, \dots, n\}$;
- (b) The color of $(i, 0)$ is 'green' for all $i \in \{1, \dots, n\}$; and
- (c) All other points on the boundary of the grid are blue.

Show that every proper 3-coloring has a unit-size square whose vertices have all three colors.

3. (Optional) Try to beat the computer in Atropos: The Sperner Game

(<http://www4.wittenberg.edu/academics/mathcomp/kburke/atropos/>).