1. Show that the set of Nash equilibria is invariant under (positive) scaling and shifting:

(a) Let $G = (\mathbf{A}, \mathbf{B})$ be a two-player game and $a, b > 0$. Then G and $G' = (a\mathbf{A}, b\mathbf{B})$ have the same set of Nash equilibria.

(b) Let $G = (\mathbf{A}, \mathbf{B})$ be a two-player game and $a, b \in \mathbb{R}$. Then G and $G' = (\mathbf{A}', \mathbf{B}')$, where

$$A'_{i,j} = A_{i,j} + a \quad \text{and} \quad B'_{i,j} = B_{i,j} + b, \qquad \text{for all } i, j,$$

have the same set of Nash equilibria.

2. Let $G = (\mathbf{A}, \mathbf{B})$ be a two-player game with all entries of $\mathbf{A}$ and $\mathbf{B}$ between 0 and 1. Show that the following algorithm always returns a $1/2$-approximate Nash equilibrium:

Step 1: Arbitrarily pick an action $i \in [m]$ for player 1;

Step 2: Given i, find an action $k \in [n]$ for player 2 that maximizes $B_{i,k}$:

$$B_{i,k} \geq B_{i,k'}, \quad \text{for all } k' \in [n];$$

Step 3: Given k, find an action $j \in [m]$ for player 1 that maximizes $A_{j,k}$:

$$A_{j,k} \geq A_{j',k}, \quad \text{for all } j' \in [m].$$

Output the following pair of distributions $(\mathbf{x}, \mathbf{y})$: $y_k = 1$ and all other entries of $\mathbf{y}$ are 0; For $\mathbf{x}$, we have the following two cases:

(i) If $i = j$, then $x_i = 1$ and all other entries of $\mathbf{x}$ are 0;

(ii) Otherwise, $x_i = x_j = 1/2$ and all other entries of $\mathbf{x}$ are 0.