1. Let A be the set of candidates with $|A| \geq 3$. Given an incentive compatible social choice function

$$f : L^n \to A$$

onto A, where L is the set of all preferences over the candidates A, we showed in class a way to construct a social welfare function

$$F : L^n \to L.$$

Show that if f is not a dictatorship function, then neither is F.

2. We have $2n$ players who live on the same street where a new hospital will be built. Each player has a secret preferred location for the hospital, denoted by $x_i \in [0, 1]$ for player i, $i \in [2n]$. If the hospital is built at $y \in [0, 1]$, then the cost of player i is

$$|x_i - y|.$$

Every player wants to minimize his or her cost. The goal is to design an *incentive compatible* mechanism *without money transfer* that can minimize the total cost

$$\sum_{i \in [2n]} |x_i - y|.$$

Consider the following natural mechanism. First we ask the players to report their preferred locations x_i, $i \in [2n]$. We order these numbers from least to greatest, and let ℓ denote the nth smallest and r denote the $(n + 1)$th smallest in the x_i's reported. Then the hospital is built at $y = (\ell + r)/2$.
It is easy to show that if all the players report truthfully their preferred locations, then the output minimizes the total cost. But is this mechanism incentive compatible? If not, give a situation where one of the players can benefit by cheating. Can you modify this mechanism and make it incentive compatible (and still minimize the total cost without money transfer)?