

1. Given a two-player game  $G = (\mathbf{A}, \mathbf{B})$ , show that  $\mathbf{x} \in \Delta_m$  is a best response w.r.t.  $\mathbf{y} \in \Delta_n$  if and only if

$$\text{For all } i, j \in [m], \text{ if } \mathbf{A}_{i,*} \mathbf{y} > \mathbf{A}_{j,*} \mathbf{y} \text{ then } x_j \text{ must be } 0,$$

where  $\mathbf{A}_{i,*}$  and  $\mathbf{A}_{j,*}$  denote the  $i$ th and  $j$ th row vectors of  $\mathbf{A}$ , respectively. (Try to use this new definition of best response in the following two problems.)

2. Consider the following two-player game  $G = (\mathbf{I}_n, -\mathbf{I}_n)$  (called Matching Pennies), where  $\mathbf{I}_n$  denotes the  $n \times n$  identity matrix.

- (a) Find a mixed equilibrium of  $G$ ; and
- (b) Prove that this mixed equilibrium is unique.

3. Consider the following two-player game  $G = (\mathbf{M}, -\mathbf{M})$  (called generalized Matching Pennies), where  $\mathbf{M}$  is a  $2n \times 2n$  matrix such that

- (i)  $M_{2i-1,2i-1} = M_{2i-1,2i} = M_{2i,2i-1} = M_{2i,2i} = 1$  for all  $i \in [n]$ ; and
- (ii) all other entries of  $\mathbf{M}$  are 0.

Show that  $(\mathbf{x}, \mathbf{y})$ , where  $\mathbf{x}, \mathbf{y} \in \Delta_{2n}$ , is a mixed equilibrium of  $G$  if and only if

$$x_{2i-1} + x_{2i} = y_{2i-1} + y_{2i} = 1/n, \quad \text{for all } i \in [n].$$