1. Given a two-player game $G = (\mathbf{A}, \mathbf{B})$, show that $\mathbf{x} \in \Delta_m$ is a best response w.r.t. $\mathbf{y} \in \Delta_n$ if and only if

$$\text{For all } i, j \in [m], \text{ if } \mathbf{A}_{i,*}\mathbf{y} > \mathbf{A}_{j,*}\mathbf{y} \text{ then } x_j \text{ must be } 0,$$

where $\mathbf{A}_{i,*}$ and $\mathbf{A}_{j,*}$ denote the ith and jth row vectors of $\mathbf{A}$, respectively. (Try to use this new definition of best response in the following two problems.)

2. Consider the following two-player game $G = (\mathbf{I}_n, -\mathbf{I}_n)$ (called Matching Pennies), where $\mathbf{I}_n$ denotes the $n \times n$ identity matrix.

(a) Find a mixed equilibrium of G; and

(b) Prove that this mixed equilibrium is unique.

3. Consider the following two-player game $G = (\mathbf{M}, -\mathbf{M})$ (called generalized Matching Pennies), where $\mathbf{M}$ is a $2n \times 2n$ matrix such that

(i) $M_{2i-1,2i-1} = M_{2i-1,2i} = M_{2i,2i-1} = M_{2i,2i} = 1$ for all $i \in [n]$; and

(ii) all other entries of $\mathbf{M}$ are 0.

Show that $(\mathbf{x}, \mathbf{y})$, where $\mathbf{x}, \mathbf{y} \in \Delta_{2n}$, is a mixed equilibrium of G if and only if

$$x_{2i-1} + x_{2i} = y_{2i-1} + y_{2i} = 1/n, \qquad \text{for all } i \in [n].$$